PreCalculus – Grade 11 and Grade 12
Curriculum Guide
August 14 – October 9, 2014
Unit 3: Functions and Analyzing Graphs

Suggested Pacing: First Nine Weeks: 4 - 90 minute class periods

Unit At A Glance:

This unit will focus mainly on the ability to analyze and describe the graphs of functions. Students will begin with the ability to model situations in a variety of ways (numerically, algebraically, and graphically). After becoming familiar with the processes and terminology, students will learn how the algebraic methods of solving relate to the graphical methods. They will then discuss the weaknesses of both and transition into an in-depth inspection of graphs. Students will learn to describe all characteristics of a graph including intercepts, asymptotes, extrema, end behavior, boundaries, continuity, and general slope. Once these descriptive factors are mastered, students will conclude the unity by applying the skills to 12 basic functions.

- In order to access all available resources, click on the hyperlinks by holding down the “Ctrl” key. Once the small hand appears, click on the mouse (you must hold the “ctrl” key as you click)

ACOS/CCRS Standards:

16. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (F-IF4)

18. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (F-IF7)
   18c. Graph rational functions, identifying zeroes and asymptotes when suitable factorizations are available, and showing end behavior. (F-IF7d)
Essential Question(s):
Essential Questions are an important component of teaching and learning which give lessons a sense of purpose and relevance. They should be designed with the intent to probe for deeper understanding of material while inciting students’ curiosity and inviting them to think critically.

- Why is it important to understand multiple methods of modeling when trying to analyze and understand data?
- How is it beneficial to be familiar with basic functions and their properties?
- How can graphs be described by their characteristics and what do these aspects reveal about the function?

Learning Objective(s):
Essential Questions are an important component of teaching and learning which give lessons a sense of purpose and relevance. They should be designed with the intent to probe for deeper understanding of material while inciting students’ curiosity and inviting them to think critically.

- I can create numerical, algebraic, and graphical models to represent a situation and its solution(s).
- I can find solutions and support them using multiple methods, and by applying the problem-solving process.
- I can recognize the weaknesses of graphing technology and avoid them by looking for hidden behavior.
- I can analyze data in order to determine if it falls into the rules of a function or a relation.
- I can determine the domain and range of a function using either the graph or the function itself.
- I can apply limits in order to describe end behavior and continuity or lack thereof.
- I can describe a graph in terms of its boundaries, maximums and minimums, output values, asymptotes, and general slope.
- I can determine if a function or its graph is odd, even, or neither.
- I know the shapes and basic properties of 12 basic functions.
**Key Vocabulary: Academic**

- analyze
- identify
- support

**Content Specific**

- asymptote
- bound
- continuity
- discontinuity
- extrema
- hidden behavior
- model
- piecewise function
- root

*The vocabulary presented in this guide do not represent all vocabulary covered by the unit. These terms are a sampling of what can be expected.

**Assessments:**

The teacher should use formative assessments to monitor students’ acquisition of knowledge while the content is being taught. By performing ongoing assessments like the ones suggested below, the teacher will have more opportunity to adjust teaching strategies, differentiate and support student learning, and predict future areas of strength and weakness. This article on Scholastic’s website contains more information on the importance of formative assessment.

- ⑥ A **Quickwrite** promotes reflection on key concepts and can serve to reinforce the vocabulary for a lesson. Allowing for exceptionally quick assessment, this strategy is ideal for discussing methods of representation and various characteristics of a graph. It is also great as an exit slip and should not take more than five minutes to complete. Some example prompts are provided below.
  - What does it mean to represent a situation numerically, algebraically, graphically, and verbally?
  - How do the different types of discontinuity affect the domain and range of a function?
  - When searching for areas of a graph that are increasing or decreasing, what specifically should be looked for algebraically?

- ⑥ **Think-Pair-Share** would allow students to collaborate on the more difficult aspects of this unit. It would especially be beneficial when learning how to analyze graphs. This strategy would be great for encouraging reflection and self-correction, while allowing students to practice multiple interpretations of a problem. By using a Think-Pair-Share to compare solving methods, students are reminded about the versatility of mathematics. An example prompt and response is provided below.
Prompt: Find the domain, range, increasing intervals, and decreasing intervals for the following graph:

Think (performed by each student alone):
Student A’s response:
Domain: \((-\infty, 1) \cup (1, \infty)\)
Range: \([-4, \infty)\)
Decreasing Intervals: \((-\infty, 0] \cup (1, \infty)\)
Increasing Intervals: \([0,1)\)

Pair (students talk about each other’s response):
Student A: “Okay, here are my answers. What are yours?”
Student B: “It looks like we got several different intervals. My domain and range do not match yours. For the domain, why did you put 1 there? It is still included because of the point here. See? It lines up with the x-axis at 1.”
Student A: “Oh I guess I did not think about that. I was just looking at those holes. I am not sure I understand this.”
Student B: “You kind of did the same thing with your range. You said the lowest part was at -4, but look at this part of the graph on the right.” etc.

Share (teacher calls on a student to share):
Student A: “I got the increasing and decreasing intervals right, but I did not understand the domain and range. Can you explain to me why the domain and range are both \((-\infty, \infty)\)? What about the holes?”

3-2-1 is a strong strategy for assessing the biggest ideas that students gathered from the lesson, while also alerting the teacher of any ideas which need more clarification. It is advised that 3-2-1 be used during lessons on determining if a representation is a function, and on lessons about discontinuity. The strategy is very versatile, but the overall goal should be to prompt students into reflecting on the day’s lesson and helping
them process what they have learned. A few example prompts are provided below:

- 3 Ways to Recognize a Function
  - 2 Examples of a Non-Function
  - 1 Question You Still Have

- 3 Types of Discontinuity
  - 2 Function Characteristics that Cause Discontinuity
  - 1 Thing I Need Clarification About

**Summative assessments** can be used to gauge students’ learning relative to content standards. They are given to determine what students have mastered. Some examples of summative assessments include, but are not limited to the following: state assessments, district benchmarks, weekly tests, end of unit tests, and end of term exams.

- **Summative Assessment I : Pre-Test August 25-September 5**
- **Post-Test October 1-15**
- **Textbook Assessments***

*Teachers should use tests provided by the textbook or manually created tests at their own discretion

**Instructional Consideration:**

**Before:**

- Data shows that writing is a weakness for the students of the district. In order to promote mathematical communication and fluency, students should be encouraged to keep a journal before or after lessons. Journaling even for a five minutes can help students grow in both confidence and ability. [Here](#) are some articles on the importance of journaling in a math classroom, along with ideas on how to incorporate it as a part of the daily routine.

- Since this unit covers a lot of the foundations needed for this courses, it is important for students to understand how to read about math. More importantly, students need to understand how to analyze and use one of their biggest resources: their textbook. Many students do not know how to use their textbooks or what their books have to offer.
Through the use of **Authentic Questioning**, the teacher can set a precedent of using textbooks as a key resource. Students can be assigned to read any section of their textbook that contains concepts for this unit. As they read, students should be instructed to stop and immediately write down any question that comes to their mind (without reading further). These questions could range from the actual material presented to the formats and symbols used within the text. Once completed, students can go back and check off questions which were answered, and share questions that they think were neglected. These questions can then be addressed until students are familiar with effective usage of their textbooks.

- For most concepts in this unit, an **Anticipation Guide** would be effective in helping students to activate their prior knowledge and experience. By having students fill one out, knowledge of vocabulary and foundational ideas will be easily and quickly assessed. This strategy is a strong tool for helping students keep track of where they started at the beginning of the unit and how much they have learned by the end of it. Here is an example anticipation guide as an [attachment](#).

  The teacher should keep in mind that this strategy can also be used during and after the lesson so that students can determine what they have learned and also reflect back upon misconceived notions. It is advised that each statement to be evaluated by the student should be defended. Instructing students to give reasons for their logic will help prevent random checking before the lessons, and will encourage students to truly reflect on their prior knowledge.

**During:**

- **Interactive Student Notebooks** are extremely effective at helping students keep their work organized and their minds engaged. Allowing for a more hands-on approach of note taking, this strategy promotes creativity while also helping students build their metacognitive abilities. One common component of interactive notebooks is called a [foldable](#). During this unit, there are several foldables which could be used to organize information. Some [example foldables](#) covering graph analysis are attached at the end of this unit.

- A **Frayer model** like the one below will allow students to thoroughly consider what they do or do not know about a subject. This strategy is useful for introducing terms and
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• **Project-based learning** does not have to involve long and complicated assignments. Instead, the focus of any project should be the ability to explore concepts learned and possibly extend beyond them. For this unit, students can be tasked with creating an illustration of some sort using the 12 basic functions and transformations of them. Students could have to provide the functions for each aspect of their graph and can be asked to perform other skills as well. The teacher should decide what skills he/she wants to assess and carefully design a rubric before assigning the project.

**Pre-AP strategies to consider**

The goal of a Pre-AP course is to prepare students for college level AP classes in the future. One distinguishable characteristic of an AP course is free response questions designed to help students delve deeper into a concept. When picking or designing problems for Pre-AP students to complete, the teacher should focus on the more challenging questions which encourage students to explore, apply, and create. For example, instead of the following problem:

What is the equation of the horizontal asymptote for the following function?

\[ f(x) = \frac{-x^2 + 3x}{x-1} \]

The teacher can consider the following as a replacement:
Consider the following function:

\[ f(x) = \frac{-x^2 + 3x}{x-1} \]

a) Where is the horizontal asymptote located?

b) What aspect of the function could be changed in order to create a point of discontinuity? Justify your answer.

A great source for any pre-AP course is the [NMSI](#) website. There are many lessons which teach students to explore, discover, and apply. A few lessons which could be applied to this unit are listed below.

- “Transforming Domain and Range” allows students to study basic transformations of an absolute value graph. Using the results, students will make a conjecture on how transformations affect the domain and range of functions.

- “Rational Functions and Their Asymptotes” encourages students to analyze the graphs of rational functions and practice finding their asymptotes. Some of the examples include graphs which cross over their horizontal asymptotes.

- “Rational Functions with Removable Discontinuities” gives students the opportunity to practice simplifying and graphing rational functions containing removable discontinuities.

**Differentiation:**

**Intervention:**

- During activities like Think-Pair-Share or Quickwrites, struggling students should be allowed to reference a list of vocabulary and their definitions.

- Students can create flash cards for the steps of calculation or creation. Then, students can be allowed to strengthen their knowledge by practicing recognition of the vocab through the processes.

- Struggling readers should be helped when trying to read through their textbook. If completing an authentic questioning in class, the teacher should take time to sit with the students and read together. Guiding the students by modeling questions that the text...
provokes should also encourage struggling readers to feel less self-conscious about their own abilities.

On Level:
- Students can write a RAFT as a more creative way of sharing what they have learned. The teacher would need to pick the Role of the writer, the Audience it is written to, the Format of what is written, and the Topic of what is created. This strategy could be used for any topic in the unit.

- Instead of utilizing the strategy of authentic questions, students can be encouraged to outline the section assigned for reading. The students should understand, however, that simply copying section titles and subtitles is not an acceptable piece of work. The outline should cover definitions and processes.

Acceleration:
- If foldables are used within the unit, excelling students should not be bound with fill-in-the-blank forms. Instead, excelling students should be encouraged to create their own forms, notes, and examples.

- Peer tutoring should be offered throughout this unit. The students who are excelling at the standards should be in charge of helping their classmates understand the material.

Accommodations: ELL/SPED
- [WWW.wida.us/standards/CAN_DOs/](http://WWW.wida.us/standards/CAN_DOs/)  
The descriptors provide a starting point for working with ELs. It serves as a tool for planning. The descriptors provide a continuum of English language development. As teachers become aware of their students English levels of proficiency, the descriptors provide sensory, graphic and interactive support needed to facilitate ELs’ access to content.

- Incorporate the following strategies: proximity seating, manipulatives/calculator, visual aids/flash cards, auditory aids/tape recorder, peer tutoring, frequent comprehension checks, repeated directions (orally) for understanding, verbal prompts, guided repetition, modeling, flow charts, cue cards with problem-solving strategies, and examples.
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Teacher Notes:

As you plan for this unit, consider the following things:

**Common misconceptions and areas of weakness**

- Basic area and perimeter formulas might need to be reviewed.

- Students are most likely going to be a little rusty when it comes to creating their own algebraic models. It might help to have students participate in Retellings when practicing these problems.

- As with the previous two units, put a strong emphasis on using the graphing calculator. Students should be so comfortable with using it that they do not need to interrupt the lesson by asking how to perform a certain task. For example, when finding a linear line of best fit, have students use their graphing calculator instead of working the problem by hand.

- Students might need a basic overview of factoring. A few reminders to consider are:
  
  - When factoring to solve a function or find its roots, do not forget to set the polynomial equal to zero first.
  
  - If the GCF is a constant, it will not affect the list of roots. The only time a GCF should be considered when finding solutions is when it contains a variable.
  
  - In order to factor trinomial quadratics whose leading coefficients are not 1, try the “bottom’s up” method. This method removes the guess-and-check nature of most other processes. [Here](#) is a video on how to do the process. This is a highly advised strategy.

- When making use of Polya’s problem-solving steps, have students label their processes in order to increase the likelihood of memorizing the method.
• For students who are still struggling to find domain and range, it is advisable to create a foldable like the one presented below:

(Source: MathEquals Love)

This foldable will increase students’ understanding of the concepts, while also easing the process of writing domains and ranges in interval notation. (Teachers can apply this strategy using sticky notes to box in domains and ranges as well. This can also be used to box in areas of increasing and decreasing intervals, or areas where f(x) < 0 and f(x) > 0.)

• Students often have a hard time identifying range when analyzing functions. This might be due to the fact that they have been trained to read graphs from left to right. Consider advising students to rotate their graph 90° clockwise. By reading the graph from left to right in this position, they might struggle less to identify the range. Another idea is to box the graph in vertically using sticky notes.

• Model logical thinking in order to help students remember how to find the vertical asymptote of a function. Remind students that a vertical asymptote is a vertical line the graph will never touch. Since a vertical line can be represented by x = #, we can conclude that a vertical asymptote is found by locating the x value which is not allowed. Students should view the function given and consider areas of concern, such as evenly indexed radicals and denominators. By understanding that these operations are bounded, students can use their properties to find the vertical asymptotes.

• Writing piecewise functions is often a difficult task for students. Advise students do some research on YouTube and find tutorials which suite their style of learning. There are vastly different approaches available.
Looking ahead

- Although this unit introduces the concept of a limit, it is not very heavily emphasized. You might want to consider setting time aside to practice the concept. There are a few calculus worksheets on Kuta Software’s website that focus on limits. Getting students accustomed to the notation now can prevent problems in the future.

For any questions, suggestions, or comments, please contact Kelli Singell at ksingell@bhm.k12.al.us

Attachments:
Anticipation Guide

Instructions: This lesson will focus on analyzing functions and their graphs. Before the lesson, carefully read each statement below and check whether you agree or disagree to the left. Provide the reason for your opinion.

During/After the lesson, re-read each statement and check whether you agree or disagree to the right. Explain why each statement is true or false at the bottom of each statement box where it says ‘Truth’.

<table>
<thead>
<tr>
<th>Before Lesson</th>
<th>Student Name: ____________________________</th>
<th>After Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agree</td>
<td>Disagree</td>
<td>Statement</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Function notation is another way of expressing an output.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reason (before):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Truth (after):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A vertical line test shows that a graph represents a function by passing through the graph more than once.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reason (before):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Truth (after):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>There are four types of discontinuity.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reason (before):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Truth (after):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The numerator of a fraction can be used to find a point of removable discontinuity.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reason (before):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Truth (after):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A graph has an increasing interval if there is a location in which its slope is positive.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reason (before):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Truth (after):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A function’s graph can cross the horizontal asymptote.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reason (before):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Truth (after):</td>
</tr>
</tbody>
</table>

Questions I would like answered during this lesson:

1. ___________________________________________________________________________________
   Was this question answered? Yes  No
   If not, did you ask the teacher? Yes  No
   What’s the answer?  ____________________________

2. ___________________________________________________________________________________
   Was this question answered? Yes  No
   If not, did you ask the teacher? Yes  No
   What’s the answer?  ____________________________
Example Foldables

(Source: http://lsquared76.wordpress.com/category/interactive-notebooks/page/4/)